Enrollment No: ______________________ **Exam Seat No:** ______________________

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name: Complex Analysis

Subject Code: 5SC01COA1 **Branch: M.Sc. (Mathematics)**

Semester : 1 **Date : 28/02/2020** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions.** **(07)**

a. Find the modulus and argument of $f(z) = -\sqrt{3} + i$. **02**

b. Write C-R equation in polar form. **02**

c. Prove that $\cos h^2 x - \sin h^2 x = 1$. **02**

d. Find real and imaginary part of $f(z) = \frac{3+2i}{1-i}$ **01**

Q-2 **Attempt all questions** **(14)**

a. State and Prove C-R equation. **07**

b. Find the complex number z if $arg(z + 2i) = \frac{\pi}{4}$ and $arg(z - 2i) = \frac{3\pi}{4}$. **05**

c. Prove that $\sin(ix) = \sin hx$ **02**

OR

Q-2 **Attempt all questions** **(14)**

a. Find the value of $\left(1 + i\sqrt{3}\right)^{90} + \left(1 - i\sqrt{3}\right)^{90}$. **05**

b. If $cosec\left(\frac{\pi}{4} + ix\right) = u + iv$ then prove that **05**

$$(u^2 + v^2)^2 = 2(u^2 - v^2)$$

c. Show that $\tanh^{-1} z = \frac{1}{2}\log\frac{1+z}{1-z}$. **04**

Q-3 **Attempt all questions** **(14)**

a. Suppose $f(z) = u + iv$ and $z_0 = x_0 + iy_0$ then $\lim_{z\to z_0} f(z) = w_0$ if and only if $\lim_{(x,y)\to(x_0,y_0)} u(x,y) = u_0$ and $\lim_{(x,y)\to(x_0,y_0)} v(x,y) = v_0$ where $w_0 = u_0 + iv_0$. **05**

b. Find the harmonic conjugate of $u(x,y) = 2x - x^3 + 3xy.^2$ **05**

c. Check the function $f(z) = \frac{1}{z}$ satisfy C-R equation or not. **04**

OR

Q-3 **Attempt all questions** **(14)**

	a.	Suppose v is harmonic conjugate of u and V is also harmonic conjugate of u then v and V are differ by constant.	**05**
	b.	Find the common roots of $z^4 + 1 = 0$ and $z^6 - i = 0$.	**05**
	c.	Suppose $f(z)$ is analytic then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2 = 4\lvert f'(z)\rvert^2$.	**04**

SECTION – II

Q-4		**Attempt the Following questions.**	**(07)**
	a.	State Residue theorem.	**02**
	b.	Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{z^n (n!)^2}{2n!}$.	**02**
	c.	Define: Analytic function	**01**
	d.	State Cauchy's theorem.	**01**
	e.	Find invariant point of the transformation $w = \frac{6z-9}{z}$.	**01**
Q-5		**Attempt all questions**	**(14)**
	a.	State and prove Cauchy's integral formula.	**07**
	b.	Evaluate $\int_c \bar{z}\, dz$ from $z = 1 - i$ to $z = 3 + 2i$ along the straight line.	**05**
	c.	State Morera's theorem.	**02**

OR

Q-5		**Attempt all questions**	
	a.	If $f(z_0) = \int_c \frac{3z^2+7z+1}{z-z_0}\, dz$ where $c: \lvert z\rvert = 2$ then find value of $f(1-i)$ and $f''(1-i)$.	**05**
	b.	State and prove M-L inequality.	**05**
	c.	Expand $f(z) = \frac{z+5}{z^2+3z+2}$ in the region $\lvert z\rvert < 1$.	**04**
Q-6		**Attempt all questions**	**(14)**
	a.	If $f(z)$ is analytic in the annular domain D given by $\rho_1 < \lvert z - z_0\rvert < \rho_2$, then $f(z)$ can be represented by the series $f(z) = \sum_{n=0}^{\infty} A_n (z-z_0)^n + \sum_{n=1}^{\infty} B_n (z-z_0)^{-n}$, where $A_n = \frac{1}{2\pi i}\int_{c_2} \frac{f(z)}{(z-z_0)^{n+1}}\, dz$ and $B_n = \frac{1}{2\pi i}\int_{c_1} \frac{f(z)}{(z-z_0)^{n+1}}\, dz$	**07**
	b.	Evaluate: $\int_c \frac{(z^2-2z)}{(z+1)^2(z^2+4)} dz$ when c is the circle $\lvert z\rvert = 3$ by using residue theorem.	**05**
	c.	Find an upper bound for the absolute value of the integral $\left\lvert \int_c e^z\, dz \right\rvert$, where c is line segment joining the point $(0,0)$ and $(1, 2\sqrt{2})$.	**02**

OR

Q-6		**Attempt all Questions**	**(14)**
	a.	Find the bilinear transformation that maps the point $0, 1, i$ in z-plane onto the points $1+i, -i, 2-i$ in the w-plane.	**05**
	b.	Find Laurent's expansion of $f(z) = \frac{7z-2}{z(z+1)(z-2)}$ in the annulus $1 < \lvert z+1\rvert < 3$.	**05**
	c.	Evaluate: $\int_c \frac{e^z}{z^2+1} dz$; where $c: \lvert z\rvert = \frac{1}{2}$.	**04**